Measurement Invariance for Second-Order Latent Growth Models: Frequentist and Bayesian Approaches

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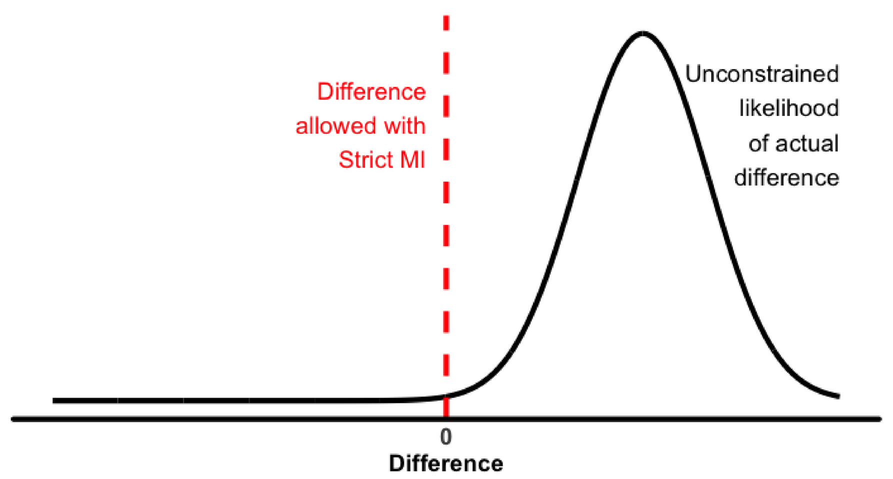
1. A Brief Introduction

- Development over time is an important focus within the social sciences. Frequently, these studies will use first-order (using observed scores) latent growth models (LGMs) to analyze their data.
- Establishing measurement invariance (MI), by which the same construct is measured in the same way at each time point, is essential for comparing scores over time. Second-order LGMs include a measurement model that can explicitly examine this.
- Current frequentist methods (full and partial scalar MI) for establishing MI are criticized for being too conservative or difficult to estimate.
- Bayesian approximate MI is an alternative that allows for some trivial deviations from full scalar MI by using zero-mean and small-variance priors. Previous research has shown that:
  - it is able to recover more accurate growth parameter estimates,
  - it provides an easy way to detect non-invariance without the use of modification indices,
  - it is easily generalized to many more groups or time points, and
  - it gets around typical MI estimation issues experienced using frequentist approaches, such as negative (residual) variances.

2. Specific Aims of This Study

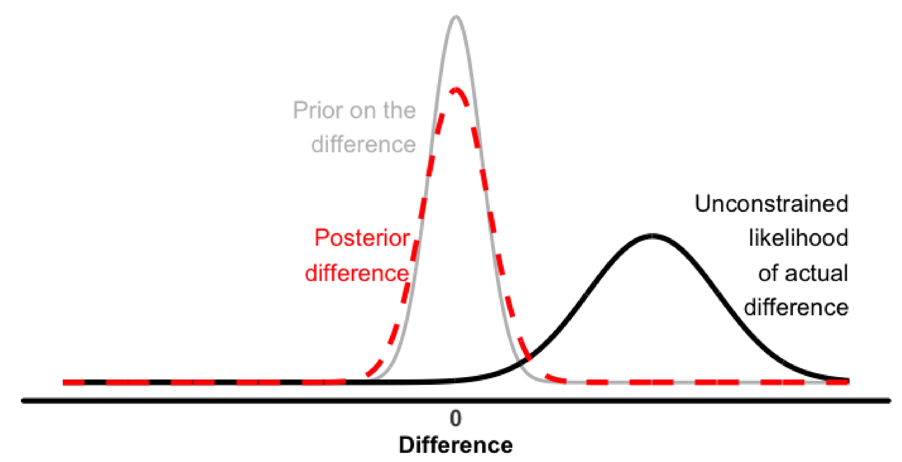
- **Aim 1.** Investigate and identify scenarios where Bayesian approximate MI improves parameter estimation over frequentist approaches.
- **Aim 2.** Investigate and identify scenarios where the Bayesian approach may not be advantageous over frequentist approaches.
  - *We predict that Bayesian approximate MI will really shine when samples are small and indicator items are categorical instead of continuous. Frequentist approaches may perform better in larger samples and with continuous indicator items.*

3. Frequentist Approach to Invariance



- This figure illustrates full scalar MI, which tests if the observed likelihood of the difference is equal to zero.

4. Bayesian Approach to (Approximate) Invariance

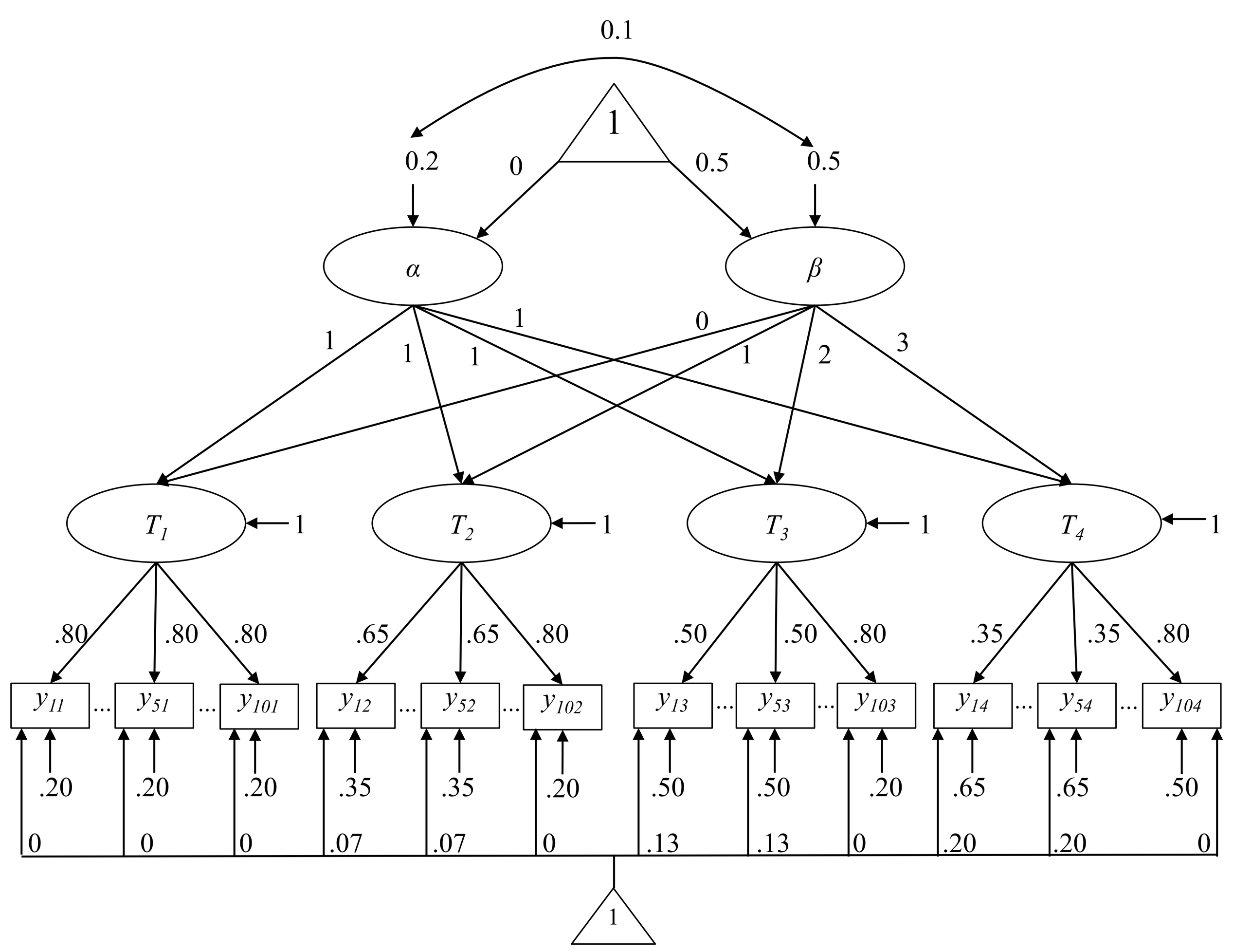


- This figure illustrates approximate MI, which uses a zero-mean and small variance prior on the difference parameter to allow some wiggle room in the posterior distribution of this difference.

5. Design

- **Estimator/Model specification:** Frequentist + Full MI, Frequentist + Partial MI, Informative Bayesian + Approximate MI (and for dichotomous indicators: Strongly Informative Bayesian + Approximate MI)
- **Sample sizes:** 100 and 500.
- **Type of indicator item:** Continuous or dichotomous.
- **Location of non-MI:** Factor loadings, intercepts/thresholds, or both.
- **Number of non-MI parameters:** 10%, 30%, or 50%.
- **Magnitude of non-MI:** Small (difference between T1 and T4 of .3 for factor loadings and .2 for intercepts) or large (difference between T1 and T4 of .45 for factor loadings and .6 for intercepts).

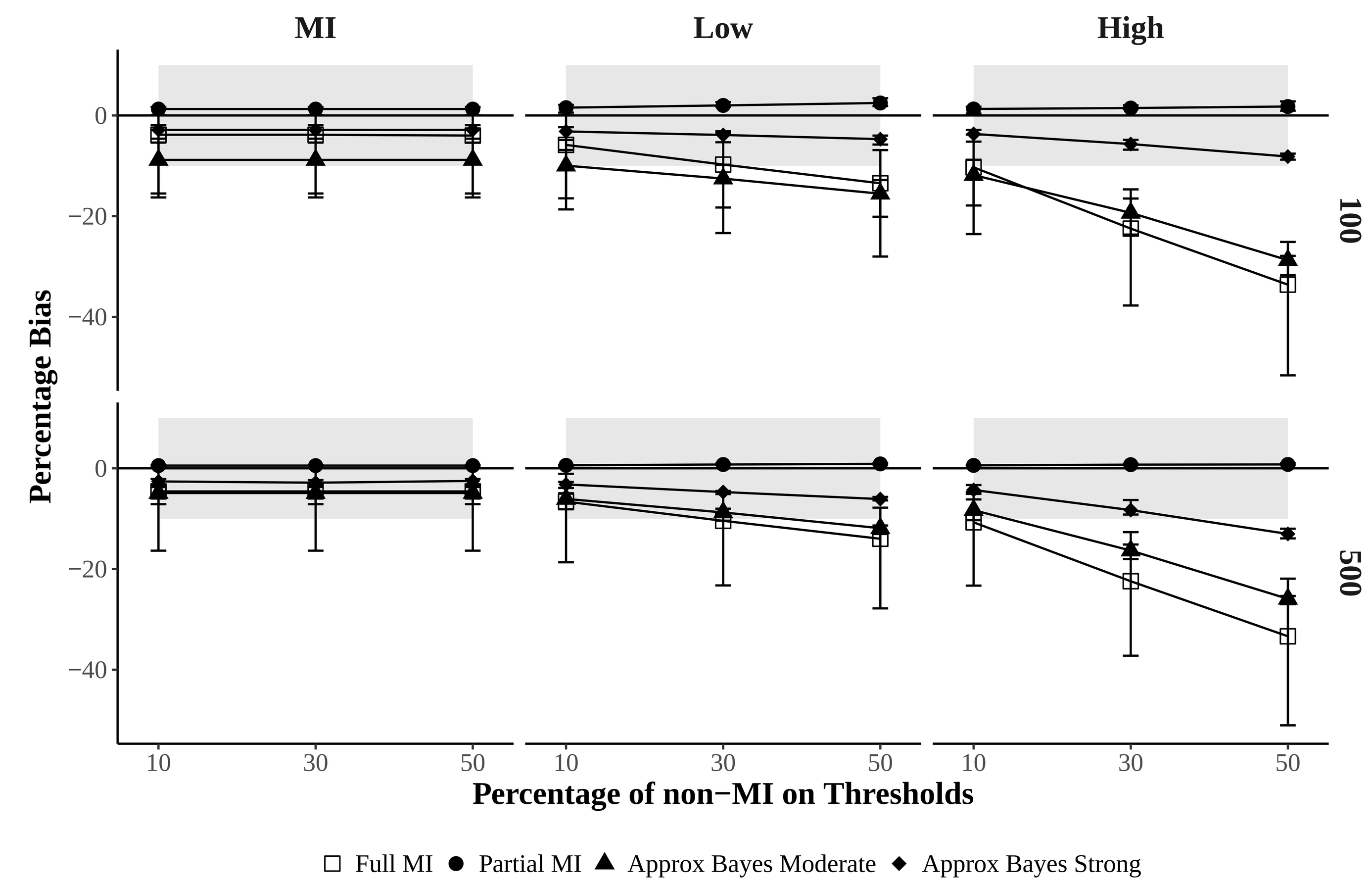
6. Model Specification

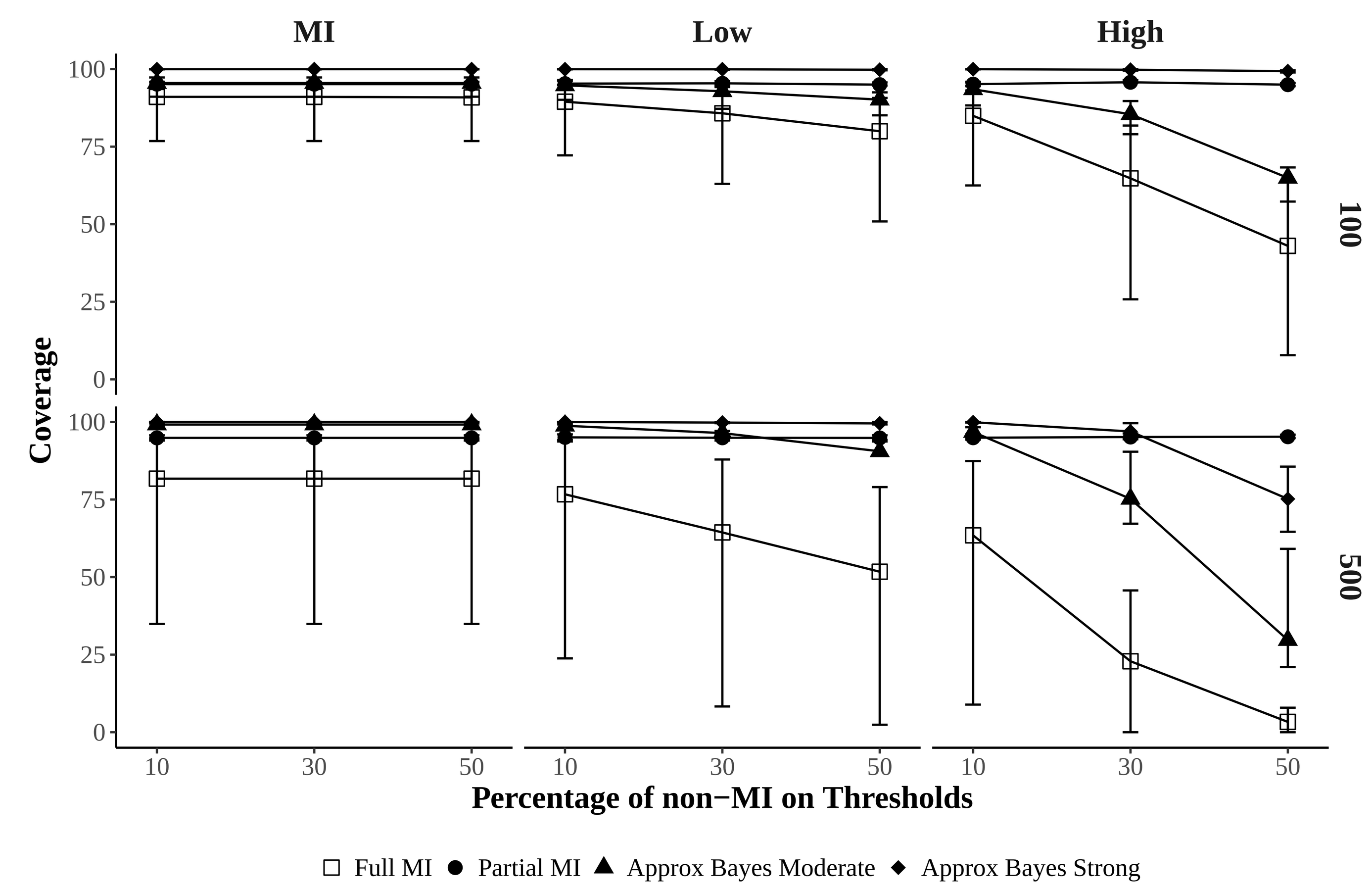


- This figure illustrates the data generating model for continuous indicators with large magnitude non-MI in 50% of loadings and small magnitude non-MI in 50% of the intercepts.

7. Selected Results

- The main parameter of interest was the mean slope estimate (population value is 0.5).
- The figures below show the performance, in terms of *relative bias and 95% CI coverage*, of the four model specifications when indicators are *dichotomous*. The *x*-axis shows the *extent of non-MI*, the three panels show the *magnitude of non-MI*, and the two rows separate the two *sample sizes* examined.





8. Implications

- **When to use frequentist methods?** When items are continuous, sample sizes are large, and the correct location of non-MI is known.
- **When to use Bayesian methods?** When items are dichotomous, sample sizes are small, the location and magnitude of non-MI are unknown and prior information is available for other parameters in the model.

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